IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 12, Issue 5 Ver. I (Sep. – Oct. 2017), PP 47-52
www.iosrjournals.org

Improve Recovery Algorithm of MWC system Based on SL0 for Multiband Signals

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Abstract: *Multiband signal is a typical signal in the realm of modern communications, whose spectrum is the sum of several narrow band signals in frequency domain. Modulated Wideband Converter (MWC) system, which is based on the emerging theory of Compressed Sensing (CS), can sample multiband signals at sub-Nyquist rate without carrier frequencies as a prior. In this paper, a novel recovery strategy for MWC is proposed, exploiting Simultaneous Smoothed Norm (SL0), to reconstruct original signal from sub-Nyquist sampling data. This method approximates norm using a continuous function, which can improve the reconstruction accuracy. Simulation results demonstrate the proposed algorithm is superior to the original orthogonal matching pursuit (SOMP) algorithm.*

Keywords: *Multiband signal; Modulated wideband converter; Compressed sensing; Simultaneous SL0*

Date of Submission: 11-09-2017 Date of acceptance: 28-09-2017

I. Introduction

Radio frequency (RF) signal is a typical signal in radar and communication systems. Generally, RF signal is modulated by high carrier frequencies in order to transform effectively. Multiband signal resides within several continuous frequency intervals spread over a wide spectrum. Therefore, it is a sparse signal that consists of a relatively small number of narrowband transmissions spread across a wide spectrum range. Wideband receiver is a representative application in communications depicted in Figure 1, in which the received signal follows the multiband model, so called multiband signal.

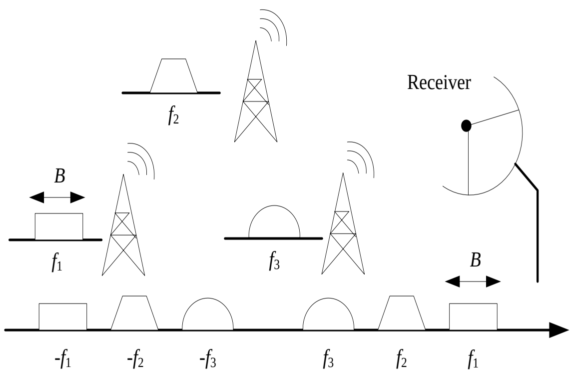


Fig 1. Three RF transmissions with different carriers f_i. The receiver sees a multiband signal.

The Nyquist sampling theorem states that analog signals can be reconstructed perfectly from their sampling data, if they are sampled at a rate that is at least twice of the highest frequency of these analog signals. Sampling multiband signals based on Nyquist theorem, however, will lead to a multitude of data, for their wideband property. A method derived by Landau [1] achieves the minimal sampling rate by demodulating each narrow band to base frequency range and then sample in Nyquist rate respectively. In paper [2], a periodic non-uniform sampling strategy was proposed as an alternative to directly sample a multiband signal at an average rate. Nevertheless, these approaches need the carrier frequencies as a prior knowledge that is always difficult to obtain.

The following papers present several sub-Nyquist strategies that have capability to treat arbitrary carrier positions: multi-coset sampling [3], the Nyquist-folding ADC [4], the random demodulator (RD) [5] and its parallel version [6], and the modulated wideband converter (MWC) [7]. These approaches, differing from each other both in sampling strategies and algorithms of recovery, process signals in different models they assume. Researches on sub-Nyquist sampling have so far focused on perfect recovery of the Nyquist-rate input signal.

This paper presents a novel recovery scheme for sub-Nyquist sampling of multiband signals based on MWC, which does not require the frequency support as a prior knowledge. Moreover, our strategy gives a better performance in recovery rate compared with SOMP. The rest of this paper is organized as follows. The theoretical background of this work is provided in Section II. In Section III, we depict the proposed algorithm in detail and the performance analysis. Section IV presents simulation results. Finally the conclusion is given in Section V.

II. Theoretical Background

A. Framework of MWC

The signal is a real-valued continuous-time signal in L2. That is signal x(t) satisfies:

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < +\infty \tag{1}$$

The Fourier transform of x(t) is defined as:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt \tag{2}$$

The framework of MWC is presented in literature [7], drawn in Fig 2.

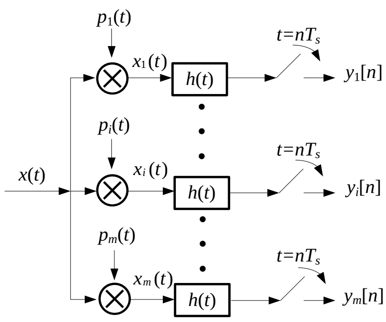


Fig 2. The modulated wideband converter-a practical sampling stage for multiband signals

where *x*(*t*) is original signal, $p_i(t)$ is a T_p-periodical mixing function, $i = 1, 2, \cdots, m$, *h*(*t*) is an ideal low-pass filter whose cutoff frequency is $1/2T_s$.

Here, a simple analysis in frequency domain for MWC is given below. Consider the i-th channel. Since pi(t) is Tp-periodic:

$$p_i(t) = \alpha_{ik}, \quad k\frac{T_p}{M} \le t \le (k+1)\frac{T_p}{M}, \quad 0 \le k \le M-1 \tag{3}$$

where $\alpha_{ik} \in \{+1, -1\}$, $p_i(t + nT_p) = p_i(t), n \in Z$, M denotes the number of ±1 intervals during each period of $p_i(t)$

The Fourier expansion of $p_i(t)$ in the *i*-th channel is

$$p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j\frac{2\pi}{T_p}lt} \tag{4}$$

where

$$c_{il} = \frac{1}{T_p}\int_0^{T_p} p_i(t) e^{-j\frac{2\pi}{T_p}lt} dt \tag{5}$$

The Fourier transform of mixed function $\tilde{x}_i(t) = x(t)p_i(t)$ is

$$\tilde{X}_i(f) = \int_{-\infty}^{\infty} \tilde{x}_i(t) e^{-j2\pi ft} dt = \sum_{l=-\infty}^{\infty} c_{il} X(f - lf_p) \tag{6}$$

$y_i[n]$ is the sampling sequence, acquired after mixing signal $\tilde{x}_i(t)$ filtered by *h*(*t*). Consequently, the discrete-time Fourier transform (DTFT) of sequence $y_i[n]$ is expressed as

$$Y_i(e^{j2\pi fT_s}) = \sum_{n=-\infty}^{\infty} y_i[n]e^{-j2\pi fnT_s} = \sum_{l=-L_0}^{+L_0} c_{il}X(f - lf_p) \tag{7}$$

Define $y_i(f) = Y_i(e^{i2\pi fT_s})$, $z_i(f) = X(f + (i - L_0 - 1)f_p)$ then it is convenient to write (7) in matrix form as

$$\mathbf{y}(f) = \mathbf{A}\mathbf{z}(f) \tag{8}$$

where $\mathbf{y}(f)$ is a vector of length m with the i-th element $y_i(f) = Y_i(e^{i2\pi fT_s})$, and $\mathbf{z}(f) = [z_1(f),...,z_L(f)]^T$, L=2L_0+1. The m×L matrix **A** contains the coefficients c_{il}.

B. Reconstruction

The key issue of recovering $x(t)$ from the sampled signals $y_i[n]$ is to determine the sparest $\mathbf{z}(f)$. The whole process is combined into a framework called a continuous to finite (CTF) system [4], depicted in Fig 3.

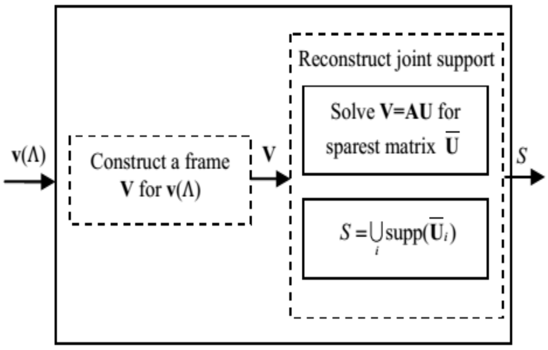


Fig 3. Continuous to finite (CTF) system

The CTF system plays a pivotal role in finding the support set S of $\mathbf{z}[n]$, where $\mathbf{z}[n]=[z_1[n],z_2[n],\cdots,z_L[n]]^T$, and $z_i[n]$ is the inverse DTFT of $z_i(f)$.

In order to use CTF, the literature [7] constructs a frame **V** for the measurement set $\mathbf{y}(\Lambda)$. Such a frame can be obtained by decomposing a matrix **Q** that is computed as follows:

$$\mathbf{Q} = \int \mathbf{y}(f)\mathbf{y}^H(f)df = \sum_{n=-\infty}^{+\infty} \mathbf{y}[n]\mathbf{y}^T[n] = \mathbf{V}\mathbf{V}^H \tag{9}$$

where $\mathbf{y}[n]=[y_1[n],y_2[n],\ldots,y_m[n]]^{\mathrm{T}}$, thus the support set S can be found through any matrix **V**. Therefore, it is possible for us to use the data in time domain directly instead of $\mathbf{y}(f)$ (in frequency domain). Note that the signal is a limited energy causal signal in practical, i.e.

$$\mathbf{Q} = \sum_{n=-\infty}^{+\infty} \mathbf{y}[n]\mathbf{y}^T[n] = \sum_{n=1}^{N} \mathbf{y}[n]\mathbf{y}^T[n] = \mathbf{V}\mathbf{V}^H \tag{10}$$

Thus we consider the sampling sequence matrix $\mathbf{y}[n]$ as the **V**, for its finite, $n = 1, 2, \cdots, N$. Once S of $\mathbf{z}[n]$ is found, $\mathbf{z}[n]$ can be reconstructed as follows:

$$\mathbf{z}_s[n] = \mathbf{A}_S^{+}\mathbf{y}[n] \tag{11}$$

$$z_i[n] = 0, \quad i \notin S \tag{12}$$

$z_i[n]$ will be interpolated to Nyquist rate, and then the original signal $x(t)$ is acquired via the inverse Fourier transform in $z_i[n]$.

As can be seen, the recovery of support set is crucial to the final signal reconstruction.

In literature [7], the support set is determined by Simultaneous Orthogonal Matching Pursuit (SOMP) [8], which is a typical algorithm in greedy pursuit, to solve underdetermined equation problems. Subspace Pursuit (SP) [9] is another greedy pursuit approach. It adopts the backtracking strategy to refine the support set, which can maintain the correct frequency supports and refine the wrong ones during next iteration. The paper [10] expands SP into Simultaneous SP and this leads to a higher success rate than SOMP. An alternative sort of approach, called linear programming such as Basis Pursuit (BP), is proposed in [11]. It is based on global optimization, while greedy pursuit is based on local optimization. Consequently, linear programming determines more accurate solution. However, the complexity of it is higher than that of greedy pursuit. In this paper, we aim at developing a novel algorithm to balance between accuracy and complexity used in CTF.

III. The Proposed Algorithm

A. Algorithm description

According to the CS theory, a naive approach to the problem of recovering **z**[n] from **y**[n] is a problem of solving the ℓ^0 norm minimization.

$$\arg\min \left\| \sum_{n=1}^{N} |\mathbf{z}[n]| \right\|_0 , \quad s.t. \quad \mathbf{y}[n] = \mathbf{A}\mathbf{z}[n] \tag{13}$$

Although there are simple recovery conditions available, the approach above is not reasonable in practice because its solution is NP-hard. Also, as ℓ^0 norm is a discontinuous function, it cannot be solved via analytical methods. A novel strategy, called Smoothed ℓ^0 Norm (SL0), has been presented in paper [12], showing that they use a continuous function to approximate the ℓ^0 norm, in order to solve the problem with analytical methods.

Consider the function

$$f_\sigma(s) = e^{(-\frac{s^2}{2\sigma^2})} \tag{14}$$

and note that

$$\lim_{\sigma \to 0} f_\sigma(x) = \begin{cases} 1, & s = 0 \\ 0, & s \neq 0 \end{cases} \tag{15}$$

define a function

$$F_\sigma(\mathbf{s}) = \sum_{i=1}^{m} f_\sigma(s_i) \tag{16}$$

where $\mathbf{s} = [s_1, \cdots, s_i, \cdots, s_m]^T$, denoting the signal vector. Then it is clear that $\|\mathbf{s}\|_0 \approx m - F_\sigma(\mathbf{s})$, and the approximation tends to be equal when $\sigma \to 0$. Thus the problem (13) is equivalent to solving

$$\arg\max F_\sigma(\mathbf{z}[n]), \quad s.t. \quad \mathbf{y}[n] = \mathbf{A}\mathbf{z}[n] \tag{17}$$

Our work is to expand SL0 to CTF system, proposing a simultaneous version of SL0 using in CTF. The procedure of the proposed algorithm is listed below.

Input: Matrix **A**, Matrix of sampling data $\mathbf{y}[n]$;
Output: The support set *S*;

Step 1. Initialization.
1.1 Solve the least square problem

$$\arg\min \|\mathbf{z}[n]\|_2 \quad s.t. \quad \mathbf{y}[n] = \mathbf{A}\mathbf{z}[n], \text{ and obtain } \hat{\mathbf{z}}^0[n]$$

1.2 Choose a suitable $\sigma_{\max} = \sigma_0 = 4\max_i \left\{ \max_n |z_i[n]| \right\}$, and $\sigma_{\min} = 10^{-5}$

Step 2. Iteration: $i = 1, 2, \cdots$
2.1 Let $\sigma_i = c\sigma_{i-1}$, $c \in [0.5, 1]$
2.2 Steepest ascent is used to solve the problem
$\arg\max F_\sigma(\mathbf{z}[n]) \quad s.t. \quad \mathbf{y}[n] = \mathbf{A}\mathbf{z}[n]$
Initialization: Set $\mathbf{z}[n] = \hat{\mathbf{z}}^{i-1}[n]$
Iteration: $j = 1, 2, 3$, $\mu = 2$

a) Let $\boldsymbol{\delta} = [z_1[n]e^{(-\frac{\|z_1[n]\|^2}{2\sigma^2})}, \quad \cdots, \quad z_n[n]e^{(-\frac{\|z_n[n]\|^2}{2\sigma^2})}]^T$
b) Let $\mathbf{z}[n] \leftarrow \mathbf{z}[n] - \mu\boldsymbol{\delta}$
c) $\mathbf{z}[n] \leftarrow \mathbf{z}[n] - \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}(\mathbf{A}\mathbf{z}[n] - \mathbf{y}[n])$

2.3 Set $\hat{\mathbf{z}}^i[n] = \mathbf{z}[n]$
2.4 If $\sigma_i < \sigma_{\min}$, then quit iteration; else go to Step 2 and continue the iteration.

Step 3. Set $\hat{\mathbf{z}}[n] = \hat{\mathbf{z}}^{I}[n]$, I is the times of iterations

Step 4. Output the support set $S = \text{supp}\sum_{n=1}^{N}|\hat{\mathbf{z}}[n]|$

For the sake of the support of $\hat{\mathbf{z}}[n]$, ℓ^1 norm of each row is used in the last step. Equivalently, any ℓ^p norm is feasible.

B. Performance analysis

Essentially, the problem about recovery of the signal is solving the ℓ^0 norm minimization problem. Compared with SOMP and SP, SL0 solves the ℓ^0 norm minimization problem directly. Furthermore, since SL0 choose σ from large to small, it is possible to avoid being trapped into local optimal solution. And this makes the algorithm more stable and possess higher recovery rate. It is more efficient for SL0 to process ℓ^0 norm using steepest ascent strategy because it approximates the ℓ^0 norm by a continuous function. And this contributes to reducing the time of processing compared with BP.

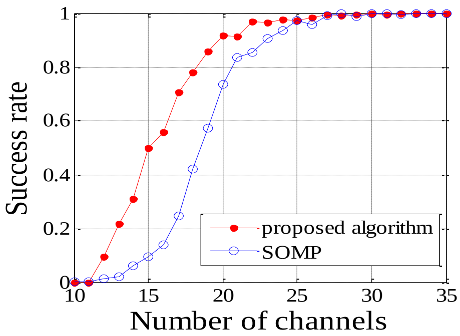
IV. Simulation Results

We will give the simulation results of our strategy comparing the original CTF. For empirical testing, we adopt the simulation strategy described in [7] for computing the success rate. The simulated signal has the following form

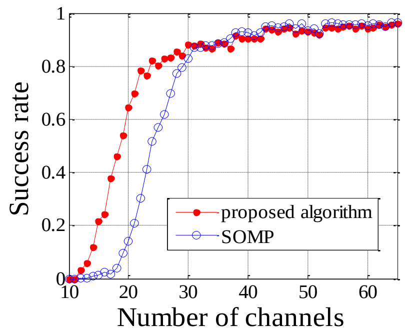
$$x(t) = \sum_{i=1}^{3}\sqrt{E_i B}\,\text{sinc}(B(t-\tau_i))\cos(2\pi f_i(t-\tau_i)) \tag{18}$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$, the original signal $x(t)$ for simulating contains i=3 pairs of rectangular bands (totally N=6), and the width of each is B=50MHz. The energy coefficients for each band is E_i={1,1,1}. Time offsets τ_i={0.4 0.7 0.3} $\mu secs$. The Nyquist rate is set f_{NYQ}=10GHz.

The number of channels m is selected from 11 to 35 for non-noisy case, while from 11 to 65 for noisy signal in our simulation experiment. We compute 500 times for each channel. Parameters of the proposed simultaneous version of SL0 are as follows. c=0.8, $\mu = 2$. Simulation results are presented in Fig 4.



(a) Noiseless signal



(b) Noisy signal. SNR=10dB

Fig 4. The simulation results-success rate of support recovery versus the number of channels

The noise mixed into the signal is white Gaussian noise. SNR is 10dB. The result of simulation demonstrates that our strategy is superior to the original CTF.

It is clear that success rate of proposed algorithm rises more steeply when channel number is relatively small, while in the large channels situation, the slope of that of SOMP is higher. This is the consequence of adding channel numbers. SOMP exploits information between channels more effectively, and this lead to the result. SL0 processes data from each channel respectively, which does not consider the interrelated information between channels.

V. Conclusion

In this paper, a strategy of recovery for MWC is proposed. Compared with the original SOMP used in CTF, our algorithm takes advantage of the high recovery rate of SL0 method. In order to find the support set more accurately, our method solve the norm minimization problem directly, which can obtain global optimal solution. Numerous simulation results confirm the conclusion that the proposed strategy shows a better performance.

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Quoc Dung Duong. "Improve Recovery Algorithm of MWC system Based on SL0 for Multiband Signals." IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE), vol. 12, no. 5, 2017, pp. 47–52.